SAMPLE QUESTION PAPER (STANDARD) - 05

Class 10 - Mathematics

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section A has 20 MCQs carrying 1 mark each
3. Section B has 5 questions carrying 02 marks each.
4. Section C has 6 questions carrying 03 marks each.
5. Section D has 4 questions carrying 05 marks each.
6. Section E has 3 case based integrated units of assessment (04 marks each) with subparts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E.
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

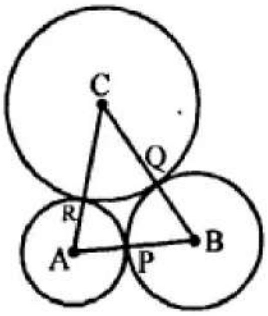
Section A

1. If $\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points A (-6 ,5) and B(-2, 3) then the value of a is **[1]**

a) 3 b) 4

c) -8 d) -4

2. In the given figure, three circles with centres A, B, C respectively touch each other externally. If AB = 5 cm, BC = 7 cm and CA = 6 cm, then the radius of the circle with centre A is **[1]**



a) 3 cm b) 1.5 cm

c) 2 cm d) 2.5 cm

3. A bag contains 50 balls of which 2x are red, 3x are white and 5x are blue. A ball is selected at random. The probability that it is not white is **[1]**

a) $\frac{7}{10}$ b) $\frac{2}{5}$

c) $\frac{7}{45}$ d) $\frac{3}{5}$

4. If points A(5, p), B(1, 5), C(2, 1) and D(6, 2) form a square ABCD, then p = **[1]**

a) 3
b) 8
c) 7
d) 6

5. The sum of two numbers is 8. If their sum is four times their difference, then the numbers are **[1]**

a) None of these
b) 7 and 1
c) 6 and 2
d) 5 and 3

6. The coordinates of the circumcentre of the triangle formed by the points O(0, 0) , A(a, 0) and B(0, b) are **[1]**

a) $\left(\frac{b}{2}, \frac{a}{2}\right)$
b) $\left(\frac{a}{2}, \frac{b}{2}\right)$
c) (b, a)
d) (a, b)

7. A school has five houses A, B, C, D and E. A class has 23 students, 4 from house A, 8 from house B, 5 from house C, 2 from house D and rest from house E. A single student is selected at random to be the class monitor. The probability that the selected student is not from A, B and C is **[1]**

a) $\frac{8}{23}$
b) $\frac{6}{23}$
c) $\frac{4}{23}$
d) $\frac{17}{23}$

8. A shuttlecock used for playing badminton is the combination of **[1]**


Shuttlecock

a) a cylinder and a hemisphere
b) frustum of a cone and a hemisphere
c) a cylinder and a sphere
d) a sphere and a cone

9. Two dice are rolled simultaneously. The probability that they get different faces on both dices is , **[1]**

a) $\frac{5}{6}$
b) $\frac{1}{6}$
c) $\frac{1}{3}$
d) $\frac{2}{3}$

10. If the equation $x^2 + 5kx + 16 = 0$ has no real roots then **[1]**

a) $k > \frac{8}{5}$
b) $k < \frac{-8}{5}$
c) $\frac{-8}{5} < k < \frac{8}{5}$
d) None of these

11. The roots of the quadratic equation $9a^2b^2x^2 - 16abcdx - 25c^2d^2 = 0$ are **[1]**

a) $\frac{25cd}{9ab}$ and $\frac{cd}{ab}$
b) $\frac{-25cd}{9ab}$ and $\frac{-cd}{ab}$
c) $\frac{-25cd}{9ab}$ and $\frac{cd}{ab}$
d) $\frac{25cd}{9ab}$ and $\frac{-cd}{ab}$

12. The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$ is **[1]**

a) $\sqrt{2}$
b) 1
c) 2
d) 0

13. If $n = 2^3 \times 3^4 \times 5^4 \times 7$, then the number of consecutive zeros in n, where n is a natural number, is **[1]**

a) 2 b) 3

c) 7 d) 4

14. The coordinates of the point which divides the join of (-6, 10) and (3, -8) in the ratio 2 : 7 is **[1]**

a) (4, -6) b) (-4, 6)

c) (1, -3) d) (-1, 3)

15. The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is **[1]**

a) $25\sqrt{3}$ b) $75\sqrt{3}$

c) 150 d) $50\sqrt{3}$

16. The mean and mode of a frequency distribution are 28 and 16 respectively. The median is **[1]**

a) 24.5 b) 24

c) 23.5 d) 22

17. The HCF and the LCM of 12, 21, 15 respectively are: **[1]**

a) 3, 140 b) 420, 3

c) 12, 420 d) 3, 420

18. In a cyclic quadrilateral ABCD, it is being given that ∠A = (x + y + 10)° ∠B = (y + 20)°, ∠C = (x + y - 30)° and ∠D = (x + y)°. Then, ∠B = ? **[1]**

a) 110° b) 70°

c) 100° d) 80°

19. **Assertion (A):** If L.C.M. {p, q} = 30 and H.C.F. {p, q} = 5, then $p \cdot q$ = 150 **[1]**
Reason (R): L.C.M. of (a, b) × H.C.F. of (a, b) = $a \cdot b$

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** In a triangle PQR, A and B are points on sides PQ and PR such that $\frac{PA}{AQ} = \frac{PB}{BR}$, then AB ∥ QR. **[1]**
Reason (R): By converse of BPT.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. There are 1000 sealed envelopes in a box, 10 of them contain a cash prize of Rs 100 each, 100 of them contain a cash prize of Rs 50 each and 200 of them contain a cash prize of Rs 10 each and rest do not contain any cash prize. If they are well shuffled and an envelope is picked up out, what is the probability that it contains no cash prize? **[2]**

22. The sum of the numerator and denominator of a fraction is 4 more than twice the numerator. If the numerator and denominator are increased by 3, they are in the ratio 2 : 3. Determine the fraction. **[2]**

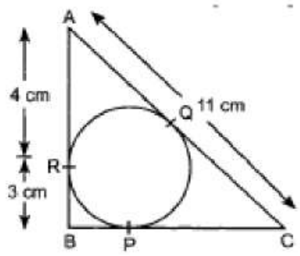
23. Find the zeroes of the polynomial $x^2 - 3$ and verify the relationship between the zeroes and the coefficients. **[2]**

24. Find the ratio in which the point P (x, 2) divides the line segment joining the points A (12, 5) and B(4, -3). Also, find the value of x. **[2]**

OR

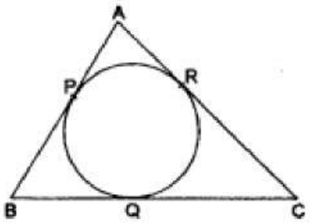
Point P divides the line segment joining the points A(2, -5) and B(5, 2) in the ratio 2 : 3. Name the quadrant in which P lies.

25. In figure, $\triangle$ABC is circumscribing a circle. Find the length of BC. **[2]**



OR

A circle is inscribed in $\triangle ABC$ touching AB, BC and AC at P, Q and R respectively. If AB = 10 cm, AR = 7 cm and CR = 5 cm, find the length of BC.



Section C

26. If (tan θ + sin θ) = m and (tan θ - sin θ) = n, prove that $(m^2 - n^2)^2 = 16mn$ **[3]**

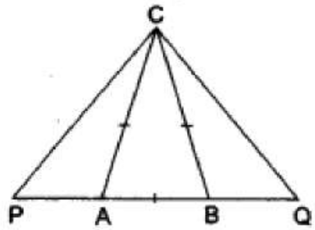
27. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are: **[3]**
y = x, y = 2x and y + x =6

28. Prove that $(3+2\sqrt{5})^2$ is irrational. **[3]**

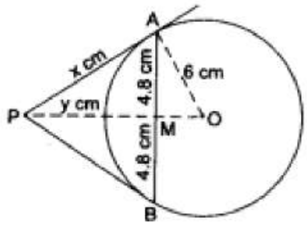
OR

Find the largest number that will divide 398, 436 and 542 leaving remainders 7,11 and 15 respectively.

29. In an isosceles, $\triangle ABC$ the base AB is produced both ways in P and Q such that $AP \times BQ = AC^2$ Prove that $\triangle ACP \sim \triangle BCQ$. **[3]**

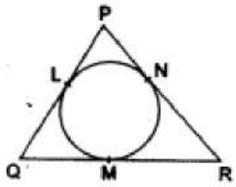


30. In the given figure, AB is a chord of length 9.6 cm of a circle with centre O and radius 6 cm. The tangents at A and B intersect at P. Find the length of PA. **[3]**



OR

In the given figure, a circle is inscribed in a triangle PQR. If PQ = 10 cm, QR = 8 cm and PR = 12 cm, find the lengths of QM, RN and PL.



31. A person observed the angle of elevation of the top of a tower is 30^o. He walked 50 m towards the foot of the tower along level ground and found the angle of elevation of the top of the tower as 60^o. Find the height of the tower. [3]

Section D

32. Solve for x: $\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}; x \neq 0, x \neq \frac{-2a-b}{2}, a, b \neq 0$ [5]

OR

If the roots of the quadratic equation $\left(c^2 - ab\right) x^2 - 2\left(a^2 - bc\right) x + b^2 - ac = 0$ in x are equal then show that either a = 0 or $a^3 + b^3 + c^3 = 3abc$

33. ABCD is a trapezium in which AB || DC and P and Q are points on AD and BC, respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD. [5]

34. Find the difference of the areas of two segments of a circle formed by a chord of length 5 cm subtending angle of 90° at the centre. [5]

OR

A semicircular region and a square region have equal perimeters. The area of the square region exceeds that of the semicircular region by 4 cm^2. Find the perimeters and areas of the two regions.

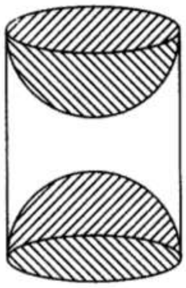
35. Calculate the mode of the following frequency distribution table : [5]

Marks	Number of students
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

Section E

36. **Read the text carefully and answer the questions:** [4]

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Find the volume of the cylindrical block before the carpenter started scooping the hemisphere from it.

(ii) Find the volume of wood scooped out?

(iii) Find the total surface area of the article?

OR

Find the total surface area of cylinder before scooping out hemisphere?

37. **Read the text carefully and answer the questions:** **[4]**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.

(i) If the piggy bank can hold 190 coins of five rupees in all, find the number of days he can contribute to put the five-rupee coins into it

(ii) Find the total money he saved.

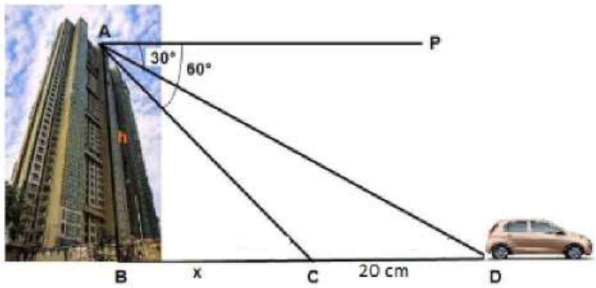
OR

How many coins are there in piggy bank on 15th day?

(iii) How much money Akshar saves in 10 days?

38. **Read the text carefully and answer the questions:** **[4]**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^o. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^o.



(i) Find the value of x.

(ii) Find the height of the building AB.

OR

Find the distance between top of the building and a car at position C?

(iii) Find the distance between top of the building and a car at position D?

Solution

SAMPLE QUESTION PAPER (STANDARD) - 05

Class 10 - Mathematics

Section A

1. **(c)** -8

   Explanation: $\frac{a}{2} = \frac{(-6-2)}{3} = -4 \Rightarrow a = -8$

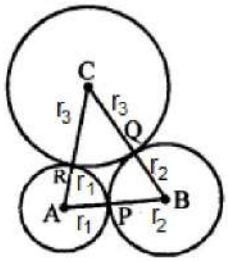
2. **(c)** 2 cm

   Explanation:

   In the given figure, three circles with centre A, B and C are drawn touching each other externally

   AB = 5 cm, BC = 7 cm and CA = 6 cm

   Let r_1, r_2, r_3 be the radii of three circles respectively

   $\therefore$AB = $r_1 + r_2$ = 5 cm ...(i)

   BC = $r_2 + r_3$ = 7 cm ...(ii)

   CA = $r_3 + r_1$ = 6 cm ....(iii)

   Adding, $2(r_1 + r_2 + r_3)$ = 18cm ---- (iv)

   Now, subtracting (ii) from(iv) respectively

   we get r_1 = 2 cm

   Hence, radius of the circle with centre A = 2 cm

3. **(a)** $\frac{7}{10}$

   Explanation: Here,

   2x + 3x + 5x = 50

   $\Rightarrow$ 10x = 50

   $\Rightarrow$ x = 5

   Number of red balls = 2 $\times$ 5 = 10

   Number of white balls = 3 $\times$ 5 = 15

   Number of blue balls = 5 $\times$ 5 = 25

   Now, Number of possible outcomes = 25 + 10 = 35

   And Number of total outcomes = 50

   $\therefore$ Required Probability = $\frac{35}{50} = \frac{7}{10}$

4. **(d)** 6

   Explanation: Vertices of a square are A(5, p), B(1, 5), C(2, 1) and D(6, 2).

   The diagonals bisect each other at O

   O is the mid-point of AC and BD

   O is mid-point of BD, then

   CO-ordinates of O will be $\left(\frac{1+6}{2}, \frac{5+2}{2}\right)$

   or $\left(\frac{7}{2}, \frac{7}{2}\right)$

   $\because$ O is mid-point of AC also

   $\therefore \frac{p+1}{2} = \frac{7}{2} \Rightarrow p + 1 = 7$

   $\Rightarrow$ p = 7 - 1 = 6

5. **(d)** 5 and 3

   Explanation: x + y = 8

x = 8 - y ... (i)

x + y = 4(x - y) ... (ii)

Substitute (i) in (ii)

8 = 4x - 4y

2 = x - y

2 = 8 - y - y

2y = 8 - 2

y = 3

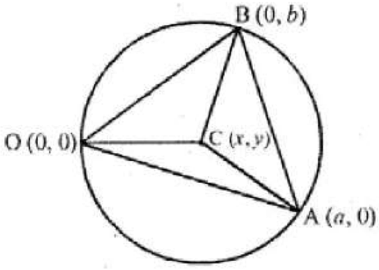
therefore, x = 8 - 3 = 5

Hence, Numbers are 5 and 3

6. **(b)** $\left(\frac{a}{2}, \frac{b}{2}\right)$

Explanation: Let co-ordinates of C be (x, y) which is the centre of the circumcircle of $\triangle$OAB

Radius of a circle are equal



$\therefore$ OC = CA = CB $\Rightarrow$ $OC^2 = CA^2 = CB^2$

$\therefore (x - 0)^2 + (y - 0)^2 = (x - a)^2 + (y - 0)^2$

$\Rightarrow x^2 + y^2 = (x - a)^2 + y^2$

$\Rightarrow x^2 = (x - a)^2 \Rightarrow x^2 = x^2 + a^2 - 2ax$

$a^2 - 2ax = 0 \Rightarrow a(a - 2x) = 0$

$\Rightarrow a = 2x \Rightarrow x = \frac{a}{2}$

and $(x - 0)^2 + (y - 0)^2 = (x - 0)^2 + (y - b)^2$

$x^2 + y^2 = x^2 + y^2 - 2by + b^2$

$\Rightarrow 2by = b^2 \Rightarrow y = \frac{b}{2}$

$\therefore$ Co-ordinates of circumcentre are $\left(\frac{a}{2}, \frac{b}{2}\right)$

7. **(b)** $\frac{6}{23}$

Explanation: Total number of students = 23

Number of students in house A, B and C = 4 + 8 + 5 = 17

$\therefore$ Remaining students = 23 - 17 = 6

So, probability that the selected student is not from A, B and C = $\frac{6}{23}$

8. **(b)** frustum of a cone and a hemisphere

Explanation: A shuttlecock used for playing badminton is a combination of a frustum of a cone and a hemisphere.

9. **(a)** $\frac{5}{6}$

Explanation: Two dice are rolled simultaneously

$\therefore$ No. of total events = 36

$\therefore$ No. of different faces can be

= 36 - (same faces) ,{Same face = (1, 1), (2, 2), (3, 3), (4, 4), (5, 5) , (6, 6) }

$\therefore$ 36 - 6 = 30

Probability P(E) = $\frac{m}{n} = \frac{30}{36} = \frac{5}{6}$

10. **(c)** $\frac{-8}{5} < k < \frac{8}{5}$

Explanation: For no real roots, we must have $b^2 - 4ac < 0$.

$\therefore \left(25k^2 - 4 \times 16\right) < 0 \Rightarrow 25k^2 < 64 \Rightarrow k^2 < \frac{64}{25} \Rightarrow \frac{-8}{5} < k < \frac{8}{5}$.

11. **(d)** $\frac{25cd}{9ab}$ and $\frac{-cd}{ab}$

Explanation: Using factorisation method

$9a^2b^2x^2 - 16abcdx - 25c^2d^2 = 0$

$\Rightarrow 9a^2b^2x^2 - 25abcdx + 9abcdx - 25c^2d^2 = 0$

$\Rightarrow abx(9abx - 25cd) + cd(9abx - 25cd) = 0$

$\Rightarrow (abx + cd)(9abx - 25cd) = 0$

$\Rightarrow abx + cd = 0$ and $9abx - 25cd = 0$

$\Rightarrow x = \frac{-cd}{ab}$ and $x = \frac{25cd}{9ab}$

12. **(b)** 1

Explanation: Given: $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)$

$= (\sec^2\theta)(1 - \sin^2\theta)$

$= (\sec^2\theta)(\cos^2\theta)$

$= \frac{1}{\cos^2\theta} \times \cos^2\theta = 1$

13. **(b)** 3

Explanation: Since, it is given that

$$n = 2^3 \times 3^4 \times 5^4 \times 7$$
$$= 2^3 \times 5^4 \times 3^4 \times 7$$
$$= 2^3 \times 5^3 \times 5 \times 3^4 \times 7$$
$$= (2 \times 5)^3 \times 5 \times 3^4 \times 7$$
$$= 5 \times 3^4 \times 7 \times (10)^3$$

So, this means the given number n will end with 3 consecutive zeroes.

14. **(b)** (-4, 6)

Explanation: Given: $(x_1, y_1) = (-6, 10)$, $(x_2, y_2) = (3, -8)$

and $m_1 : m_2 = 2 : 7$

$\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

$= \frac{2\times3+7\times(-6)}{2+7} = \frac{6-42}{9} = \frac{-36}{9}$ -4

And $y = \frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{2\times(-8)+7\times10}{2+7} = \frac{-16+70}{9} = \frac{54}{9} = 6$

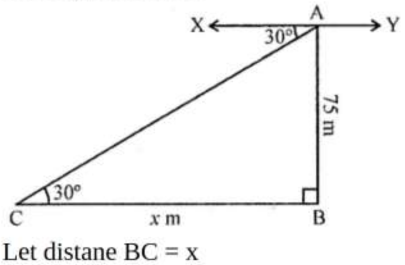
Therefore, the required coordinates are (-4, 6)

15. **(b)** $75\sqrt{3}$

Explanation: AB is as tower and AB = 75 m

From A, the angle of depression of a car C

on the ground is 30^o



Let distane BC = x

Now in right $\triangle$ACB,

$\tan\theta = \frac{AB}{BC} \Rightarrow \tan 30^o = \frac{75}{x}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{x} \Rightarrow x = 75\sqrt{3}$ m

$\therefore$ BC = $75\sqrt{3}$ m

16. **(b)** 24

Explanation: Mean = 28

Mode = 16

Mode = 3 Median - 2 Mean

Hence, $\text{Median} = \frac{\text{Mode}+2\text{Mean}}{3}$

$= \frac{16+2(28)}{3}$

$= \frac{16+56}{3}$

$= \frac{72}{3}$

= 24

17. **(d)** 3, 420

Explanation: We have,

$12 = 2 \times 2 \times 3$

$21 = 3 \times 7$

$15 = 5 \times 3$

HCF = 3

and L.C.M = $2 \times 2 \times 3 \times 5 \times 7$

= 420

18. **(d)** 80°

Explanation:

$\angle A = (x + y + 10)$, $\angle B = (y + 20)^o$, $\angle C = (x + y - 30)$ and $\angle D = (x + y)^o$

And ABCD is a cylic quadrilateral

$\Rightarrow$ Sum of opposite angles = 180^o

$\angle A + \angle B = 180^o$

$\Rightarrow x + y + 10 + x + y - 30 = 180^o$

$\Rightarrow 2x + 2y - 20 = 180^o$

$\Rightarrow 2x + 2y = 200 \Rightarrow x + y = 100$ ... (1)

And

$\angle B + \angle D = 180^o$

$\Rightarrow y + 20 + x + y = 180^o$

$x + 2y = 160^o$ .... (2)

from eqn. (1) and (2)

$x + y = 100$

$x + 2y = 160$

$-\quad -\quad -$

$+y = +60$

$\Rightarrow y = 60^o, x = 40^o$

Now $\angle B = y + 20$

$= 60 + 20 = 80^o$

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

20. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Applicability of converse of Basic Proportionality Theorem, i.e., of Thale's theorem.

Section B

21. Total number of envelopes $= 1000$

Let A = Event that envelope contains no cash

Number of envelopes containing no cash $= 1000 - (10 + 100 + 200) = 690$

$\therefore$ P(A) = $\frac{690}{100} = \frac{69}{100} = 0.69$

22. Let the numerator and denominator of fraction be x and y respectively.

Then, the fraction is $\frac{x}{y}$.

As per first condition

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator.

$x + y = 2x + 4$

$\Rightarrow -x + y = 4$........(i)

According to the second condition,

If the numerator and denominator are increased by 3, they are in the ratio 2 : 3.

$\frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow$ 3x + 9 = 2y + 6

$\Rightarrow$3x - 2y = -3.......(ii)

Multiply (i) by -2, we get

-2x + 2y = 8 .... (iii)

Adding (ii) and (iii) , we get

and 3x - 2x = -3 + 8

$\Rightarrow$ x = 5

Substituting x = 5 in (i), we get

5 - y = 4

y = 9

Hence, the required fraction is $\frac{5}{9}$

23. Given quadratic equation: $x^2 - 3$

Recall the identity $a^2 - b^2 = (a - b)(a + b)$. Using it,

we can write:

$x^2 - 3 = (x - \sqrt{3})(x + \sqrt{3})$

So, the value of $x^2 - 3$ is zero when x = $\sqrt{3}$ or x = $-\sqrt{3}$

Therefore, the zeroes of $x^2 - 3$ are $\sqrt{3}$ and $-\sqrt{3}$

Now, the sum of zeroes = $\sqrt{3} - \sqrt{3} = 0 = \frac{-(\text{ Coefficient of } x)}{\text{Coefficient of } x^2}$

and the product of zeroes = $(\sqrt{3})(-\sqrt{3}) = -3 = \frac{-3}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

24. Let P divide the line joining A and B

in the ratio of r:1

Using the section formula for the y-coordinate, we get

$2 = \frac{-3r+5}{r+1}$

$\Rightarrow$ 2r + 2 = -3r + 5

$\Rightarrow$ 5r = 3

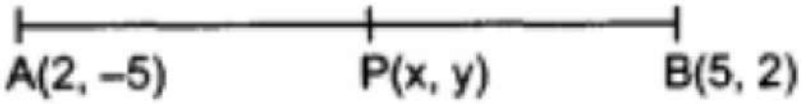
$\Rightarrow r = \frac{3}{5}$

Hence, P divides the line joining A and B in the ratio of 3:5

Using the section formula for the x - coordinate, we get

$x = \frac{12+60}{8} = \frac{72}{8} = 9$

OR



Let P(x,y) be the point which divides the points A(2, -5) and B(5, 2) in the ratio 2 : 3.

By Section formula,

(x,y) = $(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n})$

Here, x_1=2, x_2= 5, y_1=-5, y_2=2, m=2, n=3

$\Rightarrow (x, y) = (\frac{2\times5+3\times2}{2+3}, \frac{2\times(2)+3\times(-5)}{2+3})$

$\Rightarrow (x, y) = (\frac{10+6}{5}, \frac{4-15}{5})$

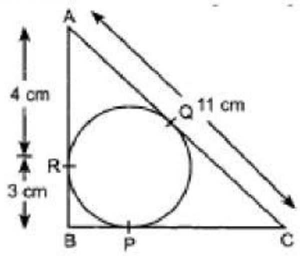
$\Rightarrow (x, y) = (\frac{16}{5}, \frac{-11}{5})$

$i.e.\ x = \frac{16}{5}$ and $y = \frac{-11}{5}$

Hence, coordinate of $P(x, y) = (3.2, -2.2)$

Now, x-coordinate is positive and y-coordinate is negative, therefore P(x,y) lies in IV quadrant.

25. Given,



$AR = 4\ cm.$

Also, $AR = AQ \Rightarrow AQ = 4cm$

Now, $QC = AC - AQ$

$= 11cm - 4cm = 7cm$ ...(i)

Also, $BP = BR$

$\therefore$ BP = 3 cm and PC = QC

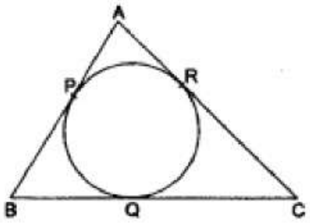
$\therefore$ PC = 7 cm [From (i)]

$BC = BP + PC$

$= 3cm + 7cm$

$= 10cm$

OR



In the given figure it is shown that a circle is inscribed in Δ ABC, such that the circle touches the sides AB, BC & AC at points P, Q & R respectively.

Also, given AR = 7 cm , CR = 5 cm , AB = 10 cm. .....(1)

We know that, tangents drawn to a circle from an external point are equal in length.

So, AP = AR, BP = BQ & CR = CQ ....(2)

From (1) & (2), AP = AR = 7 cm, CR = CQ =5 cm.....(3)

Now, from figure, BP = AB - AP = 10 - 7 = 3 cm.

$\therefore$ BP = BQ = 3cm [from (2)].....(4)

Again from figure,

BC = (BQ + QC)

$\Rightarrow$ BC = 3 + 5 [from (3) & (4)]

$\Rightarrow$ BC = 8

$\therefore$ The length of BC is 8 cm.

Section C

26. (tan θ + sin θ) = m and (tan θ - sin θ) = n

LHS = $(m^2 - n^2)^2$

$= \left[(\tan\theta + \sin\theta)^2 - (\tan\theta - \sin\theta)^2\right]^2$

$= [4\tan\theta\sin\theta]^2 \; [\because (a+b)^2 - (a-b)^2 = 4ab]$

= $16\tan^2\theta\sin^2\theta$ ...(1)

RHS = 16mn = 16 (tan θ + sin θ)(tan θ - sin θ)

$= 16\left(\tan^2\theta - \sin^2\theta\right) = 16\left(\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right)$

$= 16\left(\frac{\sin^2\theta - \sin^2\theta\cos^2\theta}{\cos^2\theta}\right)$

$= 16\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} \; [\because 1 - \cos^2\theta = \sin^2\theta]$

$= 16\frac{\sin^2\theta}{\cos^2\theta} \times \sin^2\theta$

RHS = $16\tan^2\theta\sin^2\theta$ ...(2)

$\therefore$ LHS = RHS

27. First, we form a table for all three equations different value of x and y, As:

for, y = x

x	1	2	3
y	1	2	3

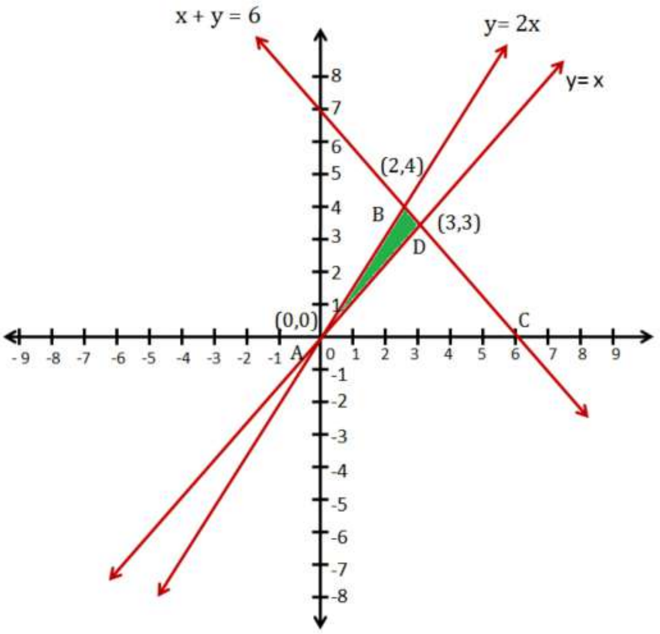
for, x + y = 6

x	6	2	3
y	0	4	3

for, y = 2x

x	1	2	3
y	2	4	6

Now we represent these points on the X-Y plane. As:



The area of the shaded region

$= A(\Delta ABC) - A(\Delta ADC)$

$= \frac{1}{2} \times$ height of $\Delta ABC \times AC + \frac{1}{2} \times$ height of $\Delta ADC \times AC$

$= \frac{1}{2} \times 4 \times 6 + \frac{1}{2} \times 3 \times 6$

= 21 cm^2

28. Let take that $3 + 2\sqrt{5}$ is a rational number.

So we can write this number as

$3 + 2\sqrt{5} = \frac{a}{b}$

Here a and b are two co-prime numbers and b is not equal to 0.

Subtract 3 both sides we get,

$2\sqrt{5} = \frac{a}{b} - 3$

$2\sqrt{5} = \frac{a-3b}{b}$

Now divide by 2 we get

$\sqrt{5} = \frac{a-3b}{2b}$

Here a and b are an integer so $\frac{a-3b}{2b}$ is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is an irrational number so it contradicts the fact.

Hence the result is $3 + 2\sqrt{5}$ is an irrational number

Now its square will again contain an irrational number.

Hence the given number is an irrational number.

OR

The largest positive integer that will divide 398, 436 and 542 leaving reminders 7, 11 and 15 respectively is the HCF of the numbers (398 – 7), (436 – 11) and (542 – 15) i.e. 391, 425 and 527.

HCF of 391,425 and 527:

HCF of 425 and 391:

425 = 391 × 1 + 34

391 = 34 × 11 + 17

34 = 17 ×2 + 0

HCF of 425 and 391 = 17

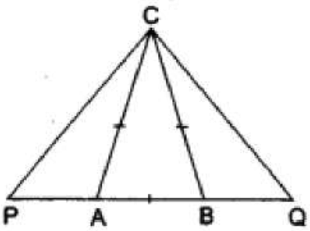
527 = 17 × 31

Similarly, HCF of 17 and 527 = 17

So, HCF of (391,425 ,527) = 17

∴ Required number is 17.

29.



It is given that $\triangle ABC$ is an isosceles triangle, therefore we have

CA = CB

$\Rightarrow \angle CAB = \angle CBA$

$\Rightarrow 180^0 - \angle CAB = 180^0 - \angle CBA$

$\Rightarrow \angle CAP = \angle CBQ$ (Angles opposite to equal sides of a triangle are equal)

Also, we have

$\mathrm{AP} \times \mathrm{BQ} = AC^2$

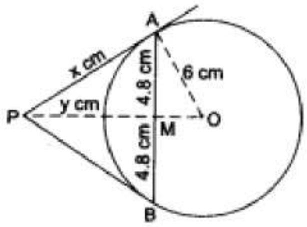
$\Longrightarrow \frac{AP}{AC} = \frac{AC}{BQ}$

$\Longrightarrow \frac{AP}{AC} = \frac{BC}{BQ}$ [∵ AC = BC]

Thus, by SAS similarity theorem, we obtain

$\triangle \mathrm{APC} \sim \triangle \mathrm{BCQ}$

30. AB is a chord of length 9.6 cm of a circle with centre O and radius 6 cm.



The tangents at A and B intersect at P.

CONSTRUCTION : Join OP and OA. Let OP and AB intersect at M.

Let PA = x cm and PM = y cm.

Now, $PA = PB$

and OP is the bisector of $\angle APB$ [∵ two tangents to a circle from an external point are equally inclined to the line segment joining the centre to that point.

Also, $OP \perp AB$ and OP bisects AB at M [∵ OP is the right bisector of AB]

∴ AM = MB = $\frac{9.6}{2}$ cm

$= 4.8cm.$

In right $\triangle AMO$, we have

$OA = 6cm$

and $AM = 4.8cm.$

∴ OM = $\sqrt{OA^2 - AM^2}$

$= \sqrt{6^2 - 4.8^2}$

$= \sqrt{12.96}$

$= 3.6\text{cm}.$

In right $\triangle PAO$, we have

$AP^2 = PM^2 + AM^2$

$\Rightarrow x^2 = y^2 + (4.8)^2$

$\Rightarrow \quad x^2 = y^2 + 23.04$ ...(i)

In right $\triangle PAO$, we have

$OP^2 = PA^2 + OA^2$ [Note $\angle PAO = 90^\circ$, since AO is the radius at the point of contact]

$\Rightarrow \quad (y + 3.6)^2$

$= x^2 + 6^2$

$\Rightarrow \quad y^2 + 7.2y + 12.96$

$= x^2 + 36$

$\Rightarrow 7.2y = 46.08$ [using (i)]

$\Rightarrow \quad y = 6.4\text{cm}$

and

$x^2 = (6.4)^2 + 23.04$

$= 40.96 + 23.04 = 64$

$\Rightarrow x = \sqrt{64} = 8$

$\therefore PA = 8cm.$

OR

According to question we are given that PQ = 10 cm, QR = 8 cm and PR = 12 cm.

We know that the lengths of the tangents drawn from an external point to a circle are equal.

Let PL = PN = x;

QL = QM = y;

RM = RN =z.

Now,PL + QL = PQ

$\Rightarrow$ x+y = 10, ...(i)

QM + RM = QR

$\Rightarrow$ y + z = 8, ...(ii)

Subtracting (ii) from (iii), we get

x - y = 4. ...(iv)

Solving (i) and (iv), we get

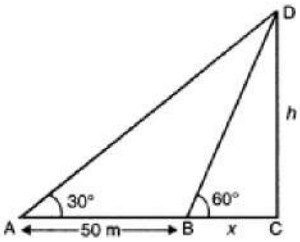
x = 7, y = 3.

Substituting y = 3 in (ii), we get z = 5

$\therefore$ QM = y = 3 cm,

RN = z = 5 cm,

PL = x = 7 cm.

31. 

Let height of the tower be DC = h m and BC = x m AC = (50 + x) m

In $\triangle$DBC, $\frac{h}{x} = \tan 60^\circ = \sqrt{3}$

$\Rightarrow \quad h = \sqrt{3}x$ ..(i)

In $\triangle$DAC, $\frac{h}{x+50} = \tan 30^\circ = \frac{1}{\sqrt{3}}$,

$\Rightarrow \quad \sqrt{3}h = x + 50$ ...(ii)

Substituting the value of h from (i) in (ii), we get

3x = x + 50

or, 3x - x = 50

$\Rightarrow$ 2x = 50

$\Rightarrow$ x = 25 m

$h = 25\sqrt{3} = 25 \times 1 \cdot 732\text{m}$

= 43.3 m

Hence, Height of tower = 43.3 m.

Section D

32. From the given equation we have:

$\frac{1}{2a+b+2x} = \frac{bx}{2bx} + \frac{2ax}{2abx} + \frac{ab}{2xab}$

$\frac{1}{2a+b+2x} = \frac{2ax+bx+ab}{2abx}$

2abx = ((2a + b) + 2x)((2a + b)x + ab)

2abx = $(2a + b)^2x + (2a + b)ab + 2(2a + b)x^2 + 2abx$

$2(2a + b)x^2 + (2a + b)^2x + (2a + b)ab = 0$

$2x^2 + (2a + b)x + ab = 0$

$x = \frac{-(2a+b)\pm\sqrt{(2a+b)^2-4\times2\times ab}}{2\times2}$

$x = \frac{-2a-ab\pm\sqrt{4a^2+b^2+4ab-8ab)}}{4}$

$x = \frac{-2(2a+b)\pm(2a-b)}{4}$

$= \frac{-2a-b-2a+b}{4}$

$x = \frac{-4a}{4} = -a$

and $x = \frac{-2a-b+2a-b}{4} = \frac{-2b}{4} = -\frac{b}{2}$

OR

We, $A = \left(c^2 - ab\right)$, B$= -2(a^2 - bc), C = b^2 - ac$

For real equal roots, $D = B^2 - 4AC = 0$

$\Rightarrow \quad \left[-2\left(a^2 - bc\right)\right]^2 - 4\left(c^2 - ab\right)\left(b^2 - ac\right) = 0$

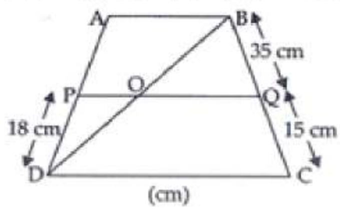
$\Rightarrow 4\left(a^4 + b^2c^2 - 2a^2bc\right) - 4\left(b^2c^2 - c^3a - ab^3 - a^2bc\right) = 0$

$\Rightarrow 4\left[a^4 + b^2c^2 - 2x^2bc - b^2c^2 + c^3a + ab^3 - a^2bc\right) = 0$

$\Rightarrow 4\left[a^4 + ac^3 + ab^3 - 3a^2bc\right] = 0$

$\Rightarrow a\left(a^3 + c^3 + b^3 - 3abc\right) = 0$

$\Rightarrow a = 0$ or $a^3 + b^3 + c^3 = 3abc$

33. 

In trapezium ABCD

AB || CD (Given)

PQ || DC (Given)

and PD = 18 cm, BQ = 35 cm and QC = 15 cm

To find: AD

∴ AB || CD || PQ ......(i)

In ΔBCD,

OQ || CD [From (i)]

$\therefore \frac{BO}{OD} = \frac{BQ}{QC}$ (ii) [By BPT]

Similarly, in ΔDAB,

PO || AB [From (i)]

$\therefore \frac{BO}{OD} = \frac{AP}{PD}$ (iii) [By BPT]

From (ii) and (iii)

$\frac{AP}{PD} = \frac{BQ}{QC}$

$\Rightarrow \frac{AP}{18} = \frac{35}{15}$

⇒ AP = $\frac{35}{15}$ × 18 = 7 × 6

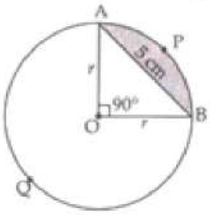
⇒ AP = 42 cm

∴ AD = AP + PD = 42 cm + 18 cm = 60 cm.

34. Chord AB = 5 cm divides the circle into two segments minor segment APB and major segment AQB. We have to find out the difference in area of major and minor segment.

Here, we are given that $\theta = 90°$

Area of $\triangle$OAB = $\frac{1}{2}$Base $\times$ Altitude = $\frac{1}{2}r \times r = \frac{1}{2}r^2$



Area of minor segment APB

$= \frac{\pi r^2\theta}{360°}$ – Area of $\triangle$AOB

$= \frac{\pi r^2 90°}{360°} - \frac{1}{2}r^2$

$\Rightarrow$ Area of minor segment = $\left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$ ...(i)

Area of major segment AQB = Area of circle – Area of minor segment

$= \pi r^2 - \left[\frac{\pi r^2}{4} - \frac{r^2}{2}\right]$

$\Rightarrow$ Area of major segment AQB $= \left[\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right]$ ...(ii)

Difference between areas of major and minor segment

$= \left(\frac{3}{4}\pi r^2 + \frac{r^2}{2}\right) - \left(\frac{\pi r^2}{4} - \frac{r^2}{2}\right)$

$= \frac{3}{4}\pi r^2 + \frac{r^2}{2} - \frac{\pi r^2}{4} + \frac{r^2}{2}$

$\Rightarrow$ Required area $= \frac{2}{4}\pi r^2 + r^2 = \frac{1}{2}\pi r^2 + r^2$

In right $\triangle$OAB,

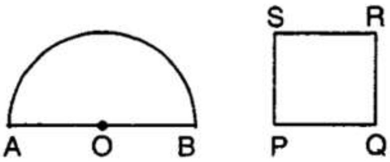
$r^2 + r^2 = AB^2$

$\Rightarrow 2r^2 = 5^2$

$\Rightarrow r^2 = \frac{25}{2}$

Therefore, required area $= \left[\frac{1}{2}\pi \times \frac{25}{2} + \frac{25}{2}\right] = \left[\frac{25}{4}\pi + \frac{25}{2}\right]$ cm^2

OR



Let radius of semicircular region be r units.

Perimeter = $2r + \pi r$

Let side of square be x units

Perimeter = 4x units.

A.T.Q, $4x = 2r + \pi r \Rightarrow x = \frac{2r+\pi r}{4}$

Area of semicircle = $\frac{1}{2}\pi r^2$

Area of square = x^2

A.T.Q, $x^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \left(\frac{2r+\pi r}{4}\right)^2 = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow \frac{1}{16}\left(4r^2 + \pi^2 r^2 + 4\pi r^2\right) = \frac{1}{2}\pi r^2 + 4$

$\Rightarrow 4r^2 + \pi^2 r^2 + 4\pi r^2 = 8\pi r^2 + 64$

$\Rightarrow 4r^2 + \pi^2 r^2 - 4\pi r^2 = 64$

$\Rightarrow r^2\left(4 + \pi^2 - 4\pi\right) = 64$

$\Rightarrow r^2(\pi - 2)^2 = 64$

$\Rightarrow r = \sqrt{\frac{64}{(\pi-2)^2}}$

$\Rightarrow r = \frac{8}{\pi-2} = \frac{8}{\frac{22}{7}-2}$ = 7 cm

Perimeter of semicircle $=2 \times 7+\frac{22}{7} \times 7$= 36 cm

Perimeter of square = 36 cm

Side of square = $\frac{36}{4}$ = 9 cm

Area of square = 9× 9 = 81 cm^2

Area of semicircle = $\frac{\pi r^2}{2}=\frac{22}{2 \times 7} \times 7 \times 7$ = 77 cm^2

35. The given data are:

Marks	**Number of students**
25 or more than 25	52
35 or more than 35	47
45 or more than 45	37
55 or more than 55	17
65 or more than 65	8
75 or more than 75	2
85 or more than 85	0

From above data we can calculate range data as following:

Marks	**Number of students(f)**
25 - 35	52 - 47 = 5
35 - 45	47 - 37 = 10
45 - 55	37 - 17 = 20
55 - 65	17 - 8 = 9
65 - 75	8 - 2 = 6
75 - 85	2 - 0 = 2
85 - 95	0

From table it is clear that maximum class frequency is 20 belonging to class interval 45 - 55

Modal class = 45 - 55

Lower limit (l) of modal class = 45

Class size (h) = 10

Frequency (f_1) of modal class = 20

Frequency (f_0) of class preceding modal class = 10

Frequency (f_2) of class succeeding the modal class = 9

Mode $= l+\left(\frac{f_1-f_0}{2f_1-f_0-f_2}\right) \times h$

$=45+\left(\frac{20-10}{2 \times 20-10-9}\right) \times 10$

$=45+\frac{10}{21} \times 10$

= 45 + 4.76

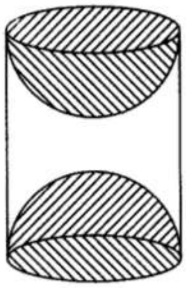
= 49.76

Therefore mode of data is 49.76

Section E

36. **Read the text carefully and answer the questions:**

A carpenter used to make different kinds and different shapes of a toy of wooden material. One day a man came to his shop to purchase an article that has values as per his requirement. He instructed the carpenter to make the toy by taking a wooden block of rectangular shape with height 12 cm and width 9 cm, then shaping this block as a solid cylinder and then scooping out a hemisphere from each end, as shown in the given figure. The difference between the length of rectangle and height of the cylinder is 2 cm (Rectangle length > Cylinder height), and the difference between the breadth of rectangle and the base of cylinder is also 2 cm (Rectangle breadth > Cylinder base(diameter)).



(i) Given:

Length of rectangle = 12 cm

Width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

Height of cylinder = 10 cm

Diameter of base = 7 cm

⇒ Radius of base = 3.5 cm

Volume of cylinder = $\pi r^2 h$

= $\frac{22}{7} \times 3.5^2 \times 10$ = 385 cm^3

(ii) Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

⇒ radius of base = 3.5 cm

Volume of wood scooped out = 2 × volume of hemisphere

⇒ Volume of wood scooped-out = $2 \times \frac{2}{3} \times \pi \times r^3$

⇒ Volume of wood scooped out = $\frac{4}{3} \times \frac{22}{7} \times (3.5)^3 = 179.66 \text{ cm}^3$

(iii) Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

⇒ radius of base = 3.5 cm

Total surface area of the article

$= 2\pi(3.5)(10) + 2\left[2\pi(3.5)^2\right]$

$= 70\pi + 49\pi = 119\pi$

$= 119 \times \frac{22}{7} = 17 \times 22$

= 374 cm^2

OR

Given:

length of rectangle = 12 cm

width of rectangle = 9 cm

After scratching the rectangle into a cylinder,

height of cylinder = 10 cm

diameter of base = 7 cm

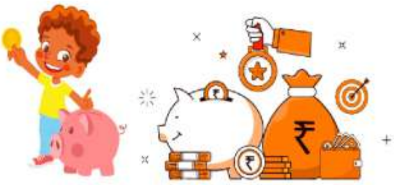
⇒ radius of base = 3.5 cm

T.S.A of cylinder = $2\pi r(r + h)$

⇒ T.S.A of cylinder = $2 \times \frac{22}{7} \times 3.5(3.5 + 10)$ = 99 cm^2

37. **Read the text carefully and answer the questions:**

Saving money is a good habit and it should be inculcated in children from the beginning. Mrs. Pushpa brought a piggy bank for her child Akshar. He puts one five-rupee coin of his savings in the piggy bank on the first day. He increases his savings by one five-rupee coin daily.



(i) Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n-1)1] = 190$

$\Rightarrow n(n + 1) = 380 \Rightarrow n^2 + n - 380 = 0$

$\Rightarrow (n + 20)(n - 19) = 0 \Rightarrow (n + 20)(n - 19) = 0$

$\Rightarrow n = -20$ or $n = 19 \Rightarrow n = -20$ or $n = 19$

But number of coins cannot be negative

$\therefore$ n = 19 (rejecting n = -20)

So, number of days = 19

(ii) Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

$= \frac{19}{2}[2 \times 5 + (19 - 1)5]$

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹950

OR

Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15-1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}$ [2 + 14]

$\Rightarrow S_{15} = 120$

So, there are 120 coins on 15th day.

(iii)Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10-1) \times 5]$

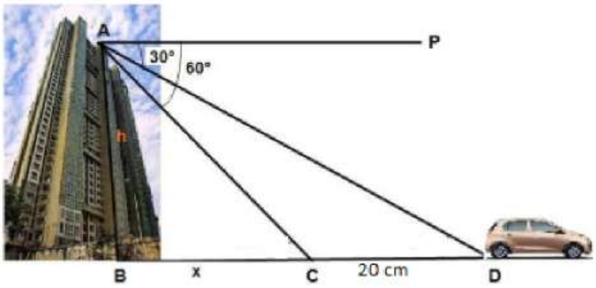
$\Rightarrow S_{10} = 5[10 + 45\}]$

$\Rightarrow S_{10} = 275$

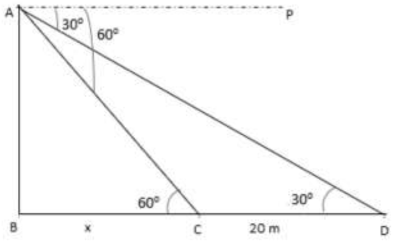
Money saved in 10 days = ₹275

38. **Read the text carefully and answer the questions:**

Vijay lives in a flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^o. After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^o.



(i) The above figure can be redrawn as shown below:



From the figure,

let AB = h and BC = x

In $\triangle$ABC,

tan 60 = $\frac{AB}{BC} = \frac{h}{x}$

$\sqrt{3} = \frac{h}{x}$

h = $\sqrt{3}$x ...(i)

In $\triangle$ABD,

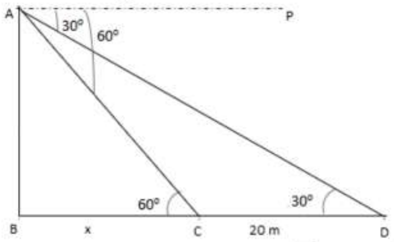
tan 30 = $\frac{AB}{BD} = \frac{h}{x+20}$

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}x}{x+20}$ [using (i)]

x + 20 = 3x

x = 10 m

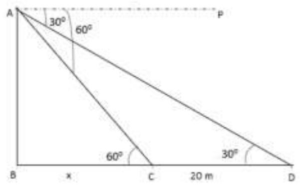
(ii) The above figure can be redrawn as shown below:



Height of the building, h = $\sqrt{3}$x = $10\sqrt{3}$ = 17.32 m

OR

The above figure can be redrawn as shown below:



Distance from top of the building to point C is

In $\triangle$ABC

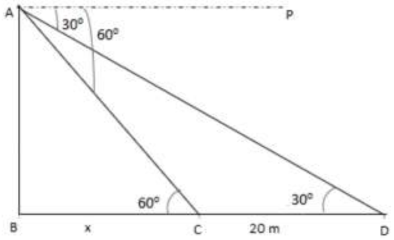
sin 60^o = $\frac{AB}{AC}$

$\Rightarrow AC = \frac{AB}{\sin 60^0}$

$\Rightarrow AC = \frac{10\sqrt{3}}{\frac{\sqrt{3}}{2}}$

$\Rightarrow$ AD = 20 m

(iii)The above figure can be redrawn as shown below:



Distance from top of the building to point D.

In $\triangle$ABD

sin 30^o = $\frac{AB}{AD}$

$\Rightarrow AD = \frac{AB}{\sin 30^0}$

$\Rightarrow AD = \frac{10\sqrt{3}}{\frac{1}{2}}$

$\Rightarrow$ AD = $20\sqrt{3}m$